Inverse Problems and Time Reversal
Inverse Problems Time Reversal
in Electromagnetics: an introduction

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Abstract

Almost every problem can be seen under two ways: the direct and the inverse one. Putting labels is (almost) a matter of convention; the easier problem is usually called "direct", letting the other term for the most difficult (and interesting) version. This is an introductory talk, assuming no special mathematical background. In a first part, inverse problems are informally defined and several simple, demystifying examples are presented [1]. Emphasis is then given to applications in electromagnetic problems. In the second part, the time reversal (TR) method [2] is presented, together with some recent results in microwave imaging [3].

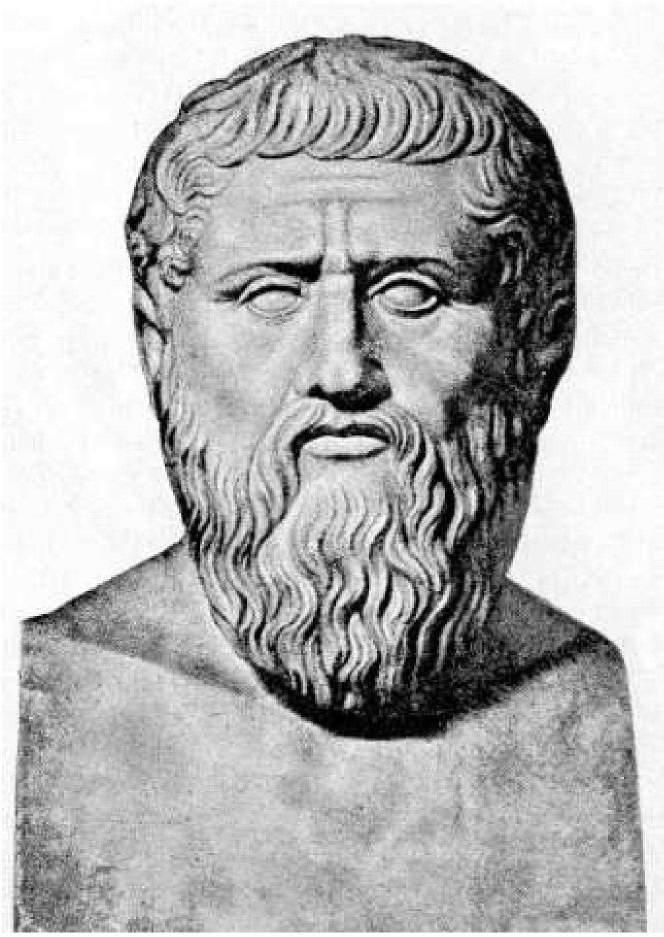
[1] Charles W. Groetsch. *Inverse problems: activities for undergraduates*. The Mathematical Association of America, Washington, DC, 1999.

[2] Mathias Fink, Didier Cassereau, Arnaud Derode, Claire Prada, Philippe Roux, Mickael Tanter, Jean-Louis Thomas, and François Wu. Time-reversed acoustics. *Reports on Progress in Physics*, 63(12):1933–1995, December 2000.

[3] Vincent Chatelée, Anthony Dubois, Ioannis Aliferis, Jean-Yves Dauvignac, Hanane El Yakouti, and Christian Pichot. Time reversal imaging techniques applied to experimental data. In *Proceedings of the Mediterranean Microwave Symposium (MMS 2006)*, Genova, Italy, September 19–21, 2006. Invited Conference.

Inverse Problems

An ancient problem...



Plato (Πλάτων) 428-348 BC

Republic VII, 360 BC: the Allegory of the Cave

Allegory of the Cave

Read the allegory for example in Wikipedia (work in progress!):
http://en.wikipedia.org/wiki/Allegory_of_the_cave

About the photo

The image is in the public domain:
http://commons.wikimedia.org/wiki/Image:Platon-2.jpg

Note

An empty slide; create shadows with hands in front of the projector!

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The allegory of the cave

Reconstruct reality from projections

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About the photo

Ice cave in Big Four Glacier, Big Four Mountain, Washington, ca. 1920
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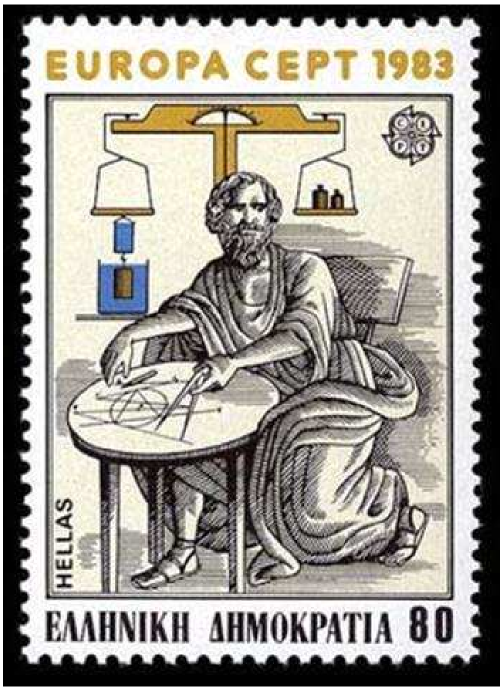
http://en.wikipedia.org/wiki/Image:Ice_cave.jpg

Early non-destructive testing. . .



Archimedes (Ἀρχιμήδης)
287-212 BC

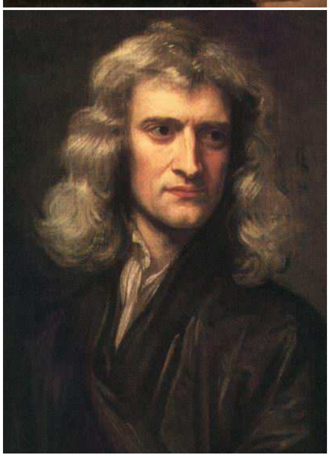
$$\text{density} = \frac{\text{mass (ok)}}{\text{volume (?)}}$$

Eureka!!! (Εὕρηκα)

About the photo

Greek postage stamp from 1983.
http://en.wikipedia.org/wiki/Image:Archimedes_greece_1983.jpg

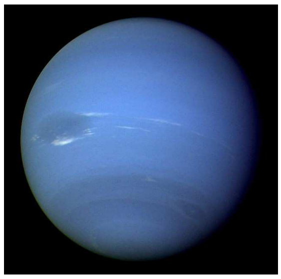
Physics' laws



- Johannes Kepler, 1571-1630
  Fit orbits to data
- Isaac Newton, 1643-1727

$$\boldsymbol{F}_{\mathbf{1\to 2}} = G\frac{m_1 m_2}{r^2}\hat{\boldsymbol{r}}_{\mathbf{2\to 1}}$$

  Fit law(s) to orbits
- Le Verrier, 1811-1877
  Fit planets to laws (1846)

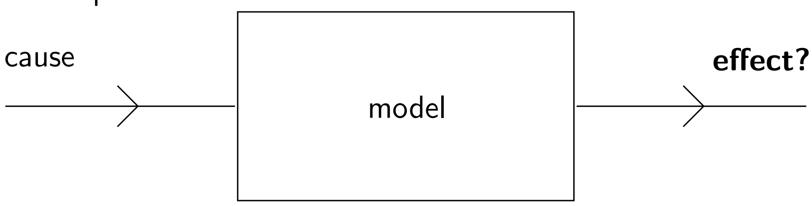


About the photos

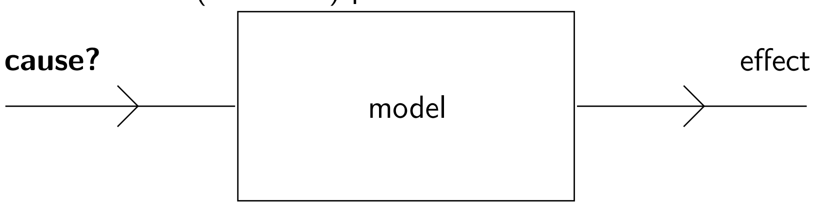
J. Kepler: image in the public domain
http://en.wikipedia.org/wiki/Image:Johannes_Kepler_1610.jpg
I. Newton: image in the public domain
http://en.wikipedia.org/wiki/Image:GodfreyKneller-IsaacNewton-1689.jpg
Neptune by Voyager2: image in the public domain
http://en.wikipedia.org/wiki/Image:Neptune.jpg

A problem never comes alone

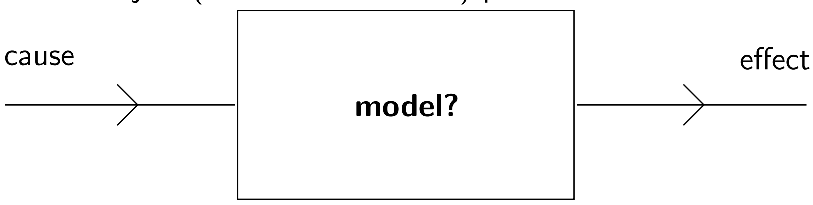
- Direct problem



- Inverse source (causation) problem



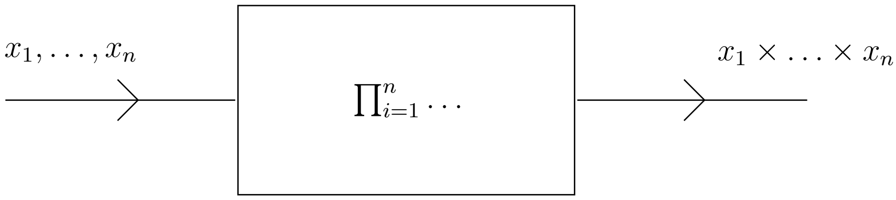
- Inverse object (model identification) problem



- One direct + two inverse problems!

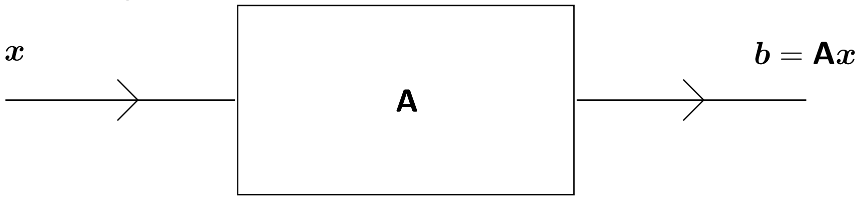
Two examples

- Simple math



Direct (multiplication): $3 \times 3 \times 67 \times 389 = 234567$
Inverse source (factorization): $234567 = ?$

- Linear Algebra



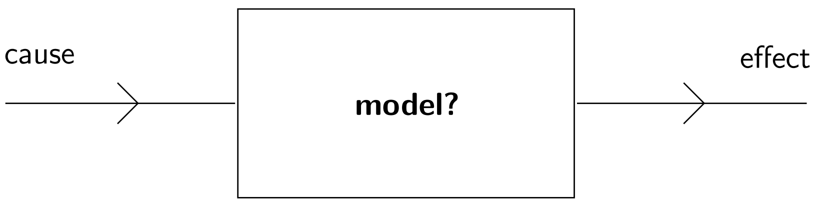
Direct: matrix-vector multiplication
Inverse source: matrix inversion $\boldsymbol{x} = \mathbf{A}^{-1}\boldsymbol{b}$

- Direct: easy / Inverse: difficult

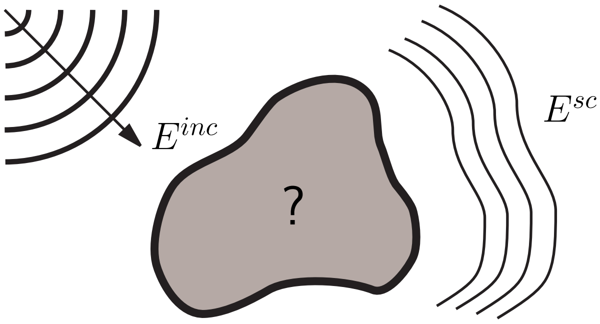
An ill-posed problem

- Direct problems: “no problem” (well posed)
- Inverse problems: at least one *is not* true:

1. The solution exists
2. The solution is unique
3. The solution is continuous with respect to data

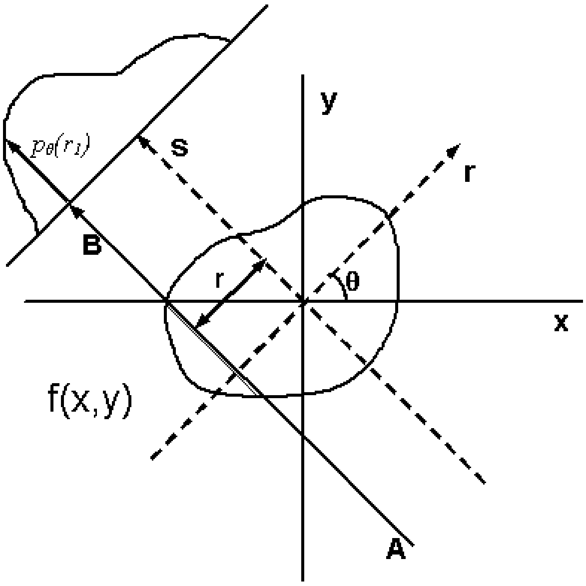


Inverse problems in electromagnetics



- Data: incident and scattered field
- Unknowns: object properties (2D or 3D)
  - Qualitative: detection/shape
  - Quantitative: shape and $\varepsilon(\boldsymbol{r}), \sigma(\boldsymbol{r})$

Qualitative imaging: beyond Radon



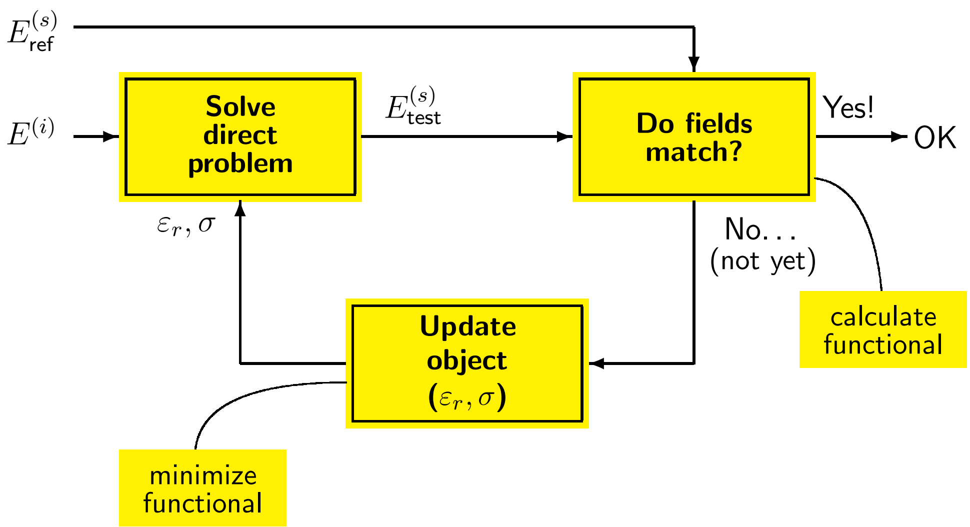
$$\mathcal{R}[f](r, \theta) = \iint f(x, y)\delta(x\cos\theta + y\sin\theta - r)\,\mathrm{d}x\,\mathrm{d}y$$

Reconstruct object from projections (line integrals)

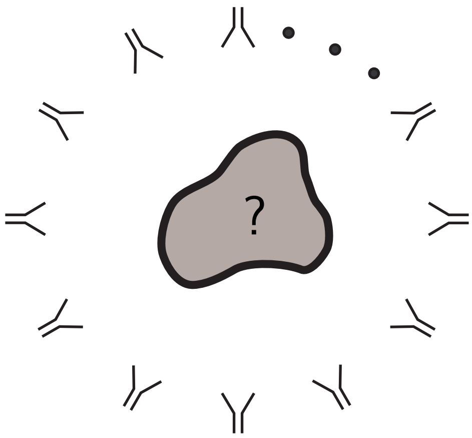
Note

Works for rectilinear propagation (X-rays).
The drawing is under the GNU Free Documentation License.
http://en.wikipedia.org/wiki/Image:Tomographic_fig1.png

Quantitative imaging: iterative algorithm

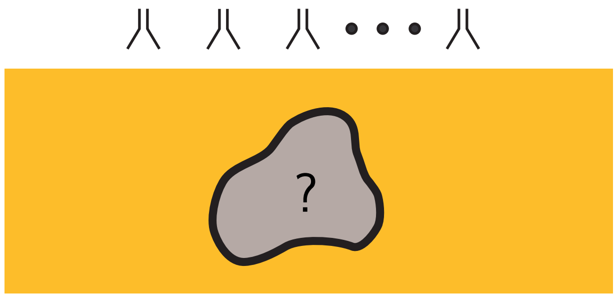


Free space imaging



√ Emitters/receivers surround the object

Imaging of buried objects



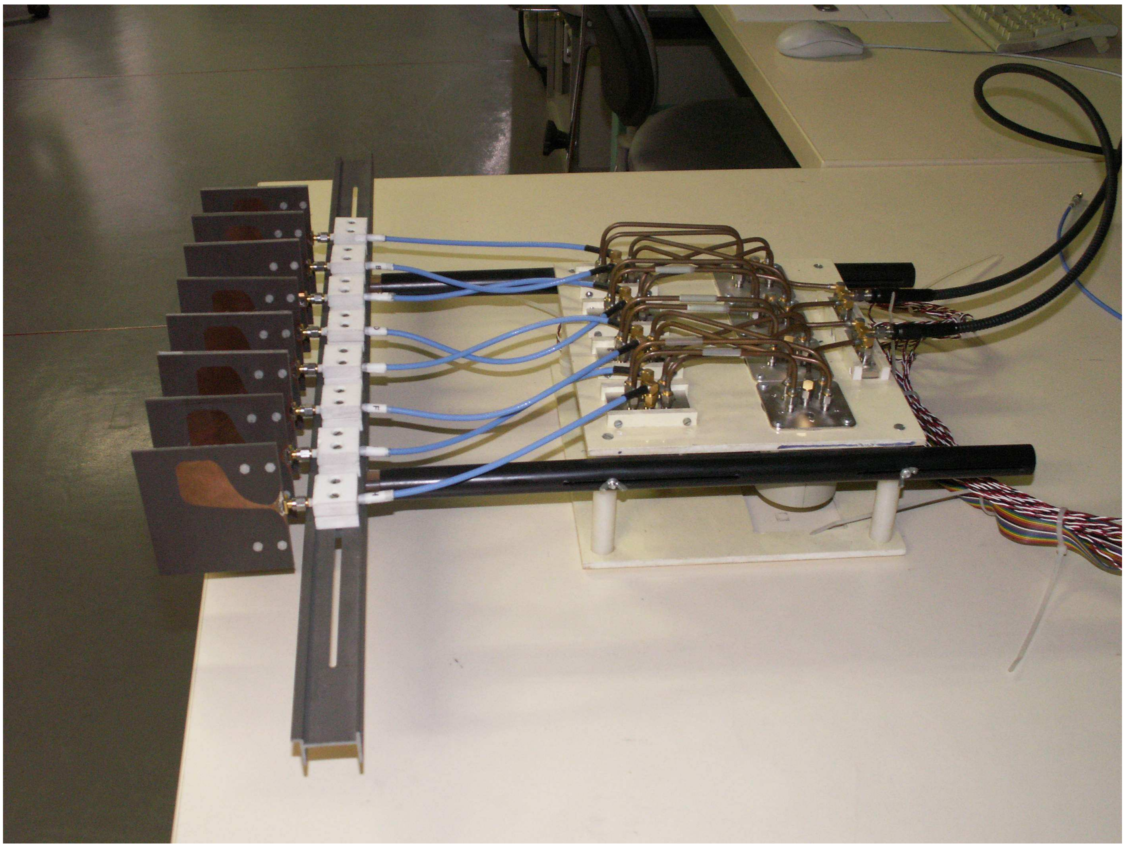
× Limited scattering data

Time Reversal

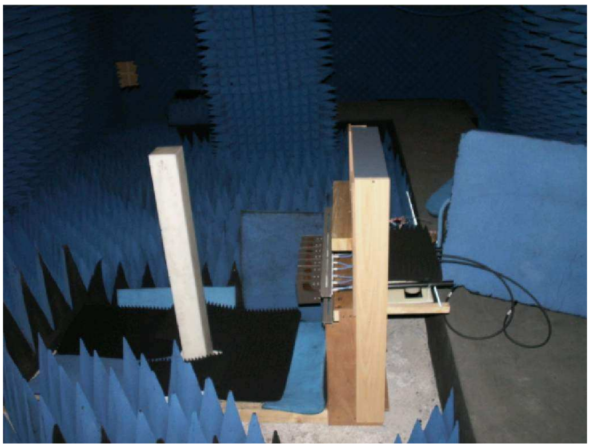
Time reversal techniques

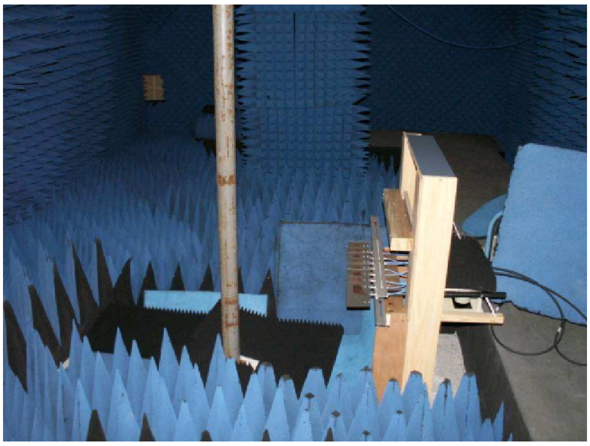
1. TR: Time-Reversal (time domain)
   - ▼ Wave equation: invariable under $t \to -t$ (lossless media)
   - ▼ Reciprocity: interchange emitter/receiver (far field)
   - ▼ Receivers record $f(t)$
   - ▼ Receivers emit* $f(-t)$: videotape rewind
   - * simulation: determine wave past; experiment: focus on scatterer(s)
2. DORT: Decomposition of the Time Reversal Operator (frequency domain)
   - ▼ $\mathbf{K} = \begin{pmatrix} S_{11} & \dots & S_{1N} \\ \dots & \dots & \dots \\ S_{N1} & \dots & S_{NN} \end{pmatrix}$ $\quad N \times N$ complex matrix
   - ▼ $\mathbf{K}$: emitters $\longrightarrow$ receivers
   - ▼ $\mathbf{K}^{\dagger}$: receivers $\longrightarrow$ emitters
   - ▼ $\mathbf{L} = \mathbf{K}^{\dagger}\mathbf{K}$ time reversal operator

Synthetic-Impulse Microwave Imaging System



The “unknown” scatterers



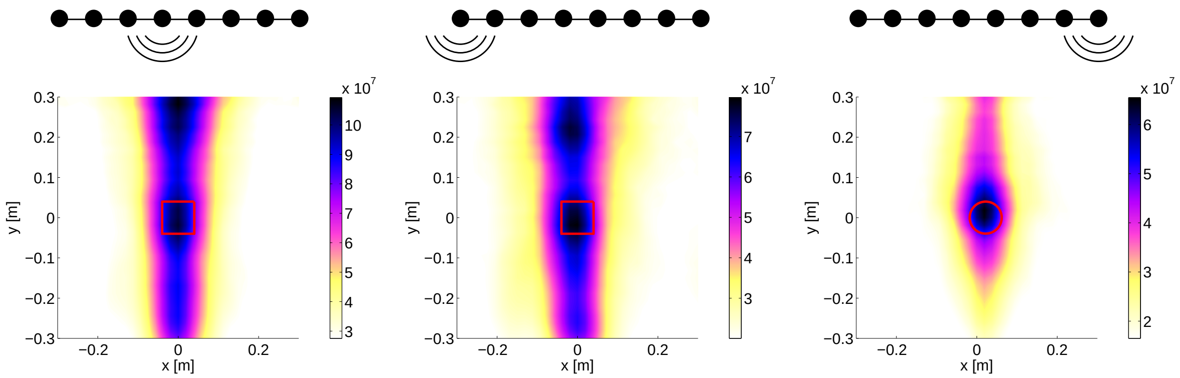


- Dielectric, $\varepsilon_r = 3$
- Square cross-section
- Size: 10 cm

- Metallic
- Circular cross-section
- Diameter: 7 cm

TR on real data

- Target at 80 cm, inter-antenna distance 5 cm; line length 35 cm
- $A(\boldsymbol{r}) = \int |E_z(\boldsymbol{r}, t)|^2 \, \mathrm{d}t$: accumulated energy during the “film”



dielectric target

metallic target

Comment

Shown: cumulative energy over the time of a synthetic TR experiment. We indicate the antenna used in emission (real data received by all the other antennas). The received data are TR'ed and radiated (simulation) to create focusing in the target.

Applications

- Medical imaging
- Demining
- Civil engineering
- Non destructive testing
- Antenna optimization
- Security
- ...

Activities

- Theory
  - Electromagnetic forward/inverse scattering
  - Optimization/Regularization techniques
  - Level Sets
  - Time Reversal / DORT
- Algorithms
  - Computational EM: MoM, FDFD, etc.
  - 2D/3D inversion, metallic/dielectric objects
- Systems
  - Antenna design
  - Data aquisition:
    Synthetic Impulse Microwave Imaging System (SIMIS)