



Interval Analysis & Robust System Solving under Uncertainty

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Interval arithmetic

- Outer evaluation of the range of f

$$\begin{aligned}x \in [1, 2] \quad y \in [3, 4] &\longrightarrow (x + y) \in [4, 6] \\&\longrightarrow \sin(x) \in [0.84147, 1] \\&(\sin(1) = 0.841470984808)\end{aligned}$$

- The *dependence effect*

$$x - x \in [-1, 1]$$

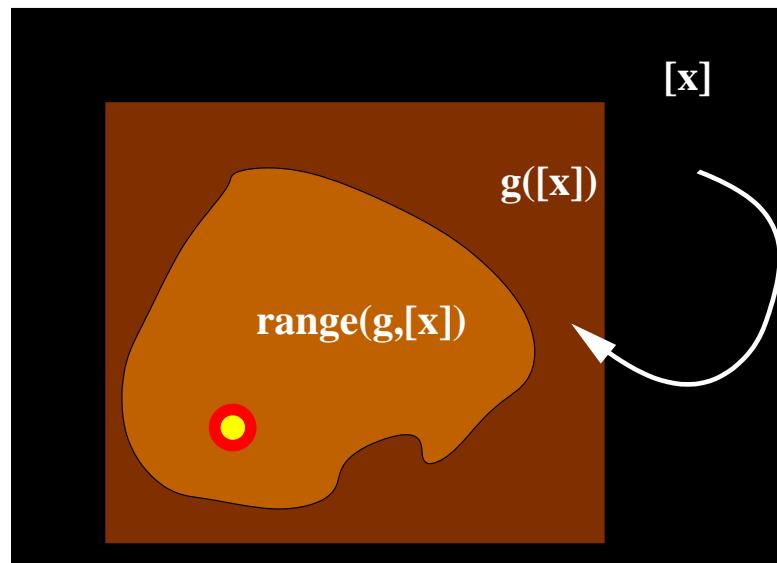
- Convergence issue

$$\lim_{\text{rad}([x]) \rightarrow 0} \text{rad}(f([x])) = 0$$

An example of certification

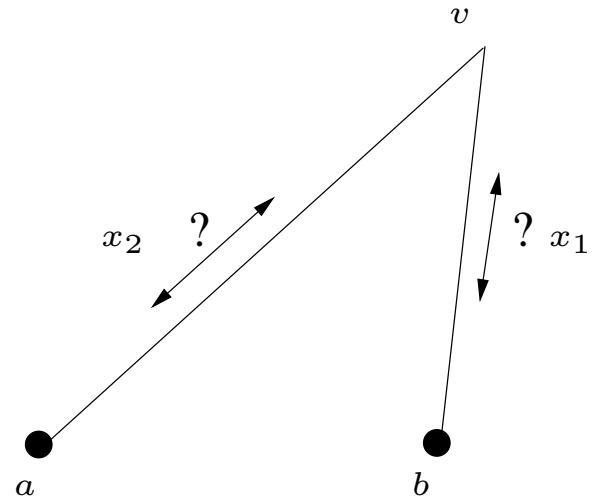
■ Brouwer's theorem

$$f(x) = 0 \iff g(x) - x = 0 \iff x = g(x)$$



Example : workspace of a parallel robot

$$(v_1 - a_1)^2 + (v_2 - a_2)^2 = x_1^2$$
$$(v_1 - b_1)^2 + (v_2 - b_2)^2 = x_2^2$$



$$\Sigma := \{x \in [x] \mid f(x) = 0\}$$

$$(v_1 - a_1)^2 + (v_2 - a_2)^2 - \textcolor{red}{x_1}^2 = 0$$

$$(v_1 - b_1)^2 + (v_2 - b_2)^2 - \textcolor{red}{x_2}^2 = 0$$

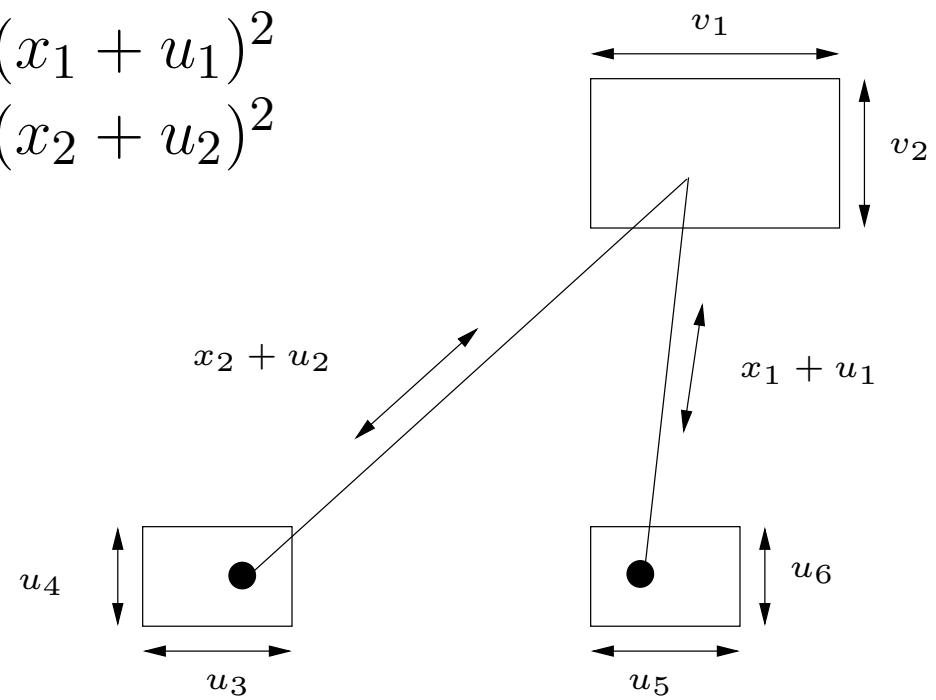
$$(\textcolor{red}{v}_1 - a_1)^2 + (\textcolor{red}{v}_2 - a_2)^2 - x_1^2 = 0$$

$$(\textcolor{red}{v}_1 - b_1)^2 + (\textcolor{red}{v}_2 - b_2)^2 - x_2^2 = 0$$

Example : workspace of a parallel robot

Robustness under uncertainty

$$(v_1 - u_3)^2 + (v_2 - u_4)^2 = (x_1 + u_1)^2$$
$$(v_1 - u_5)^2 + (v_2 - u_6)^2 = (x_2 + u_2)^2$$

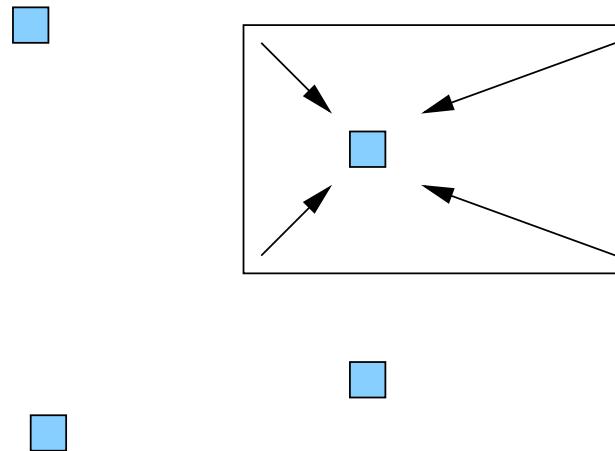


→ AE-solution set

$$\Sigma := \{x \in [x] \mid (\forall u \in [u]) (\exists v \in [v]) f(u, v, x) = 0\}$$

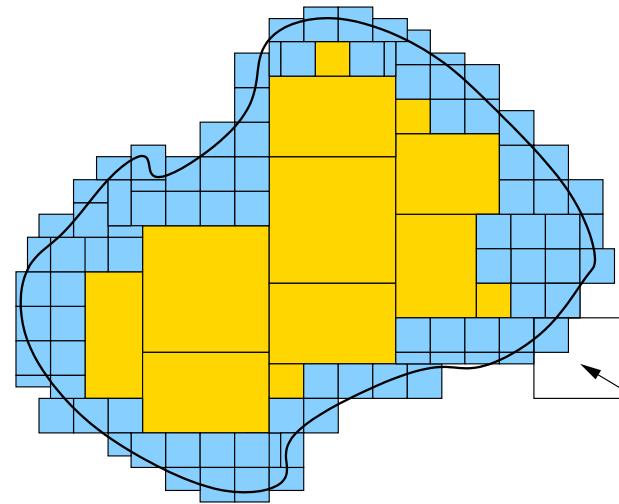
Branch & Bound

$$f(x) = 0$$



Branch & Bound with AE-solution sets

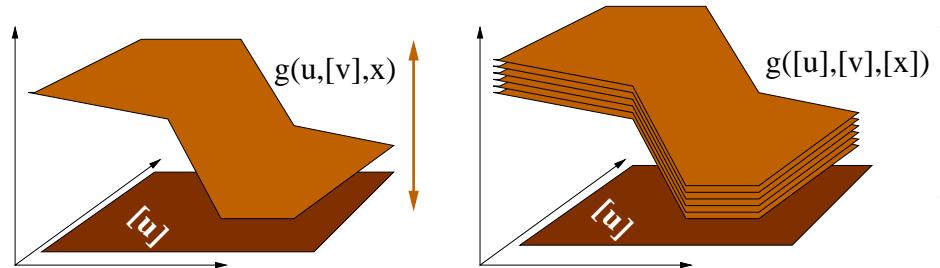
$$f(u, v, x) = 0$$



Example of inner test

$$f(u, v, x) = 0?$$

$$f(u, v, x) = v - g(u, v, x)$$
$$g([u], [v], [x]) \subset [v] \implies \text{Brouwer's theorem!}$$



$$(\forall x \in [u]) (\forall u \in [u]) (\exists v \in [v]) \quad f(u, v, x) = 0$$

$[x]$ is an **inner box**.