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Table of Contents

Modeling Cortical Activity: Cortical Columns

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Abstract. The aim of the talk is to introduce some notions about cortical modeling with a focus on cortical columns. In the first part, I introduce cortical columns from a biological point of view and point out why the notion of cortical column is complex. In the second part, I describe Jansen's model of a single cortical column and discuss its adequacy to biological facts. The third and last part shows how a system describing a single cortical column, like Jansen's, can be inserted in a model of cortical sheet, with the purpose of modeling large scale cortical activity.

1 Cortical Columns from the Biological Point of View

It has been hypothesized that small vertical structures called *cortical columns* are the basic units of sensory and motor information processing in the cortex [1]. How can such a structure emerge from the complexity of the cortex?

Many cortical neurons throw their axons and dendrites from the cortex surface to the white matter thereby forming the anatomical basis of the columnar organization in the cortex.

In 1957, Mountcastle discovered a columnar organization in the cortex. With electrode recordings, he showed that neurons inside columns of 300 to 500 μm of diameter displayed similar activities. Those physiological units are usually called *macrocolumns*. Some of them are spatially well defined while some others are more difficult to distinguish from one another. What is the meaning of such units?

Many experiments on somatosensory and visual cortices made it possible to relate physiological columns with sensory functions. In some cases the processing site for a given function is clearly defined like in rat's sensory cortex where every whisker is associated with a sharply bounded cortical site in layer IV. In other cases, the information processing sites move continuously across the surface of the cortex when stimulation varies so that it is not possible to define a size for columns. It is the case of the *orientation columns* in the primary visual cortex [2].

2 Jansen's Model of a Single Cortical Column

The model features a population of pyramidal neurons that receive excitatory and inhibitory feedback from inter-neurons residing in the same column and an

excitatory input from other columns and sub-cortical structures like the thalamus [3]. Writing the equations describing information processing inside and between the different populations leads to a six-dimensional dynamical system driven by its input parameter p, representing the strength of sensory stimulation.

We performed a bifurcation analysis of the system (*i.e.*, a study of the behaviors the system can display according to different values of a parameter) according to p and showed that it is able to produce essentially two types of activity: alpha activity (an oscillatory activity around 10 Hz) and spikes like those recorded on epileptic patients.

What critics can we make on this model? First, it might be too simplistic because it reduces a hundred of thousands neurons with complex interactions to three populations. Another problem arises from the impossibility to validate a model of a single cortical column experimentally because there is no recording of the activity of such an object isolated from the rest of the brain.

3   Towards Large Scale Cortical Modeling: Continuum of Cortical Columns

We need a new framework to study cortical columns. Since they cannot be isolated from the cortex and we want to be able to validate our model, we should simulate a continuum of columns accounting for the entire cortex, or just several areas of it, and compare the results to large scale recordings of cortical activity obtained from MEEG (Magneto- and Electroencephalography). Besides, the idea that columns form a rigid network of well separated units appears to be false from a biological point of view. Functional columns have been shown to overlap and one should rather consider that there is a cortical column under every point of the cortical surface. These ideas led us to reformulate the equations of Jansen's model and include a spatial dimension in its variables. We obtain a system of integro-differential equations in a Banach space for which we can show local existence and unicity of the solution. The numerical method for solving it relies on Picard-Lindelöf's theorem. At each time instant t, we can write the activity of the cortex as a fixed point of a known operator and obtain an approximation of it by Picard iterations.

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An Introduction to Sand Automata

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Abstract. In this talk we present sand automata, a new model for simulating various phenomena. We introduce it as an extension of existing models, leaving room for a more complete study.

1 Introduction

Sandpile models are widely used for simulating natural phenomena that consist in moving particles. For instance, an interesting formal model named *Sand Pile Model* (SPM) for the simulation of sandpiles has been introduced and studied in [1]. The simplicity of its formalization contrasts with the complexity of its dynamical behavior.

The issue is that all these results cannot be easily generalized. For this reason, the classical discrete dynamical systems point of view has been studied through a new model, sand automata [2,3]. Their formal definition is similar to cellular automata [4], with the additional constraint that modifications on a configuration should obey some consistency rule.

We introduce progressively these models and a few interesting results on the dynamics of sand automata.

2 Definitions

SPM A *configuration* (*sandpile*) is a sequence of integers $c = (c_1, \ldots, c_l)$, where for all $1 \leq i \leq l$, $c_i \in \mathbb{N}$ is the number of grains in the ith column. SPM evolves according to a very simple local rule : a grain falls from a column i to its right neighbor $i + 1$ if $c_i \geq c_{i+1} + 2$.

SPM has fixed point dynamics (*i.e.* after some transient time, nothing happens), its behavior is precisely described in [1].

Cellular Automata *Cellular automata* (CA) [4] are a common discrete dynamic system which acts on configurations with the help of a local rule. In dimension 1, a *configuration* c is a bi-infinite sequence of states $c \in S^{\mathbb{Z}}$ (S is a finite set of states).

The *local rule* is applied to every point of the configuration at the same time. It changes a state into a new one according to the states of a fixed number of

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[⋆] Joint work with Enrico Formenti.

neighbors. For example, in the simplest case, we only consider the current cell and its left and right neighbors. In this case, the local rule $\rho : S \times S \times S \mapsto S$ returns a new state depending on the state of the three cells.

CA are a much more complex model, widely studied from different points of view (simulation, languages, calculability, *etc.*).

Sand Automata *Sand automata* (SA) [2] are a king of "hybrid" of sand pile models and cellular automata. We mix the unbounded states of SPM with the infinite length of the configurations of CA. In a similar way to CA, all cells are updated synchronously, using a local rule (common to CA and SPM).

Formally, a *configuration c* is now a point of $\widetilde{\mathbb{Z}}^{\mathbb{Z}}$, where $\widetilde{\mathbb{Z}} = \mathbb{Z} \cup \{-\infty, +\infty\}$ is the set of states (the number of grains). The local rule transforms this configuration, adding or removing grains from every column at the same time, depending on a bounded neighborhood.

It is easy to simulate a CA with a SA, and *vice-versa*. The main difference between the two models is the fact that in a sand configuration, there can be no "holes" between sand grains. Moreover, this is clearly a generalization of SPM: not only can grains move, but they can also disappear or be created anywhere, provided the local rule allows it.

3 Research Interests

Once it has been properly formalized, this new model provides new possibilities for studying the simulated phenomena. We are mainly interested in the long-term dynamics: given a particular SA, what can we say about it?

(i) Which initial conditions will create a particular configuration (*bijectivity*)?
(ii) Is the total amount of grains preserved along the evolution of the system (*grain conservation*)?
(iii) Starting from any configuration, will it reach a periodic state (*stabilization* of the pile)?

These problems are difficult, partially or totally solved in [3]. Point (i) is still an open question, while point (ii) is proved to be decidable and (iii) is undecidable.

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A Very Short Introduction to Interval Analysis

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Interval analysis is a branch of numerical analysis, devoted to dealing with the accuracy issue of computer-based calculations. It all started in the late sixties with the seminal book by Moore [1]. Further well-known reference books on this topic are [2–5].

The goal of interval analysis is to design methods that cope with all kind of imprecision that hinders classical numerical techniques from providing reliable results. Such imprecision can model rounding errors as well as data uncertainties inherent to real-life problems.

The basic idea of interval analysis is to embed intervals that include the range of all possible error made, in any low-level computation. Therefore, computations are performed with the so-called *interval arithmetics*, that takes interval operands instead of real operands, e.g.,

$$[1,2] + [2,3] = [3,5].$$

Indeed, if $x \in [1,2]$ and $y \in [2,3]$, then one can see that $x + y$ can only lie in $[3,5]$. Interval computations are also possible with elementary functions such as *exp*, *sqr*, or *sin*, based on our knowledge of their monotonicity properties. A more complex function f can also be evaluated recursively on an interval vector $\mathbf{x}$ (also called **a box**), as long as the expression of f is a chain of elementary functions. However, this only results in a (sometimes rather crude) outer approximation of the range of f on $\mathbf{x}$. Will shall write

$$f(\mathbf{x}) \supset range(f, \mathbf{x}). \tag{1}$$

So far, interval analysis has been mostly used to solve systems of equations. For that purpose, the method combines interval arithmetics with a combinatorial search to find all the solutions in a given initial domain. Let f be a mapping from $\mathbb{R}^n$ to $\mathbb{R}^m$, and $\mathbf{x}_0$ be an interval vector. We can enforce the following procedure to find all the solutions of $f(x) = 0$ in $\mathbf{x}_0$:

> push $\mathbf{x}_0$ on a stack
> **while** the stack is not empty **do**
>   pop a box $\mathbf{x}$ from the stack
>   **if** width($\mathbf{x}$) $< \epsilon$ **then**
>   store $\mathbf{x}$ as a potential solution
>   **else if** $0 \in f(\mathbf{x})$ **then**
>   split $\mathbf{x}$ into two interval vectors $\mathbf{x}_1$ and $\mathbf{x}_2$
>   push $\mathbf{x}_1$ and $\mathbf{x}_2$ on the stack
>   **end if**
> **end while**

The soundness of this algorithm relies on (1). Indeed, if $0 \notin f(\mathbf{x})$, then we know that no solution exist in $\mathbf{x}$, so that this box can be safely discarded. Unless they are removed by the latter test, boxes are split until their width get smaller than a user-defined precision ϵ. The sharper the evaluation of f, the more likely a box can be removed, and clearly, this has a direct consequence on both the overall efficiency and accuracy. This is why devising sharp evaluations has been a crucial matter in interval analysis.

Existence of Solutions

Each box $\mathbf{x}$ returned by the algorithm could not be proven to be infeasible, but, so far, nothing proves that it does contain a solution. Some techniques exist to guarantee the existence of a solution in $\mathbf{x}$. For instance, we can avail ourselves of Brouwer's theorem. This theorem states that any continuous function of a compact set to itself has a fixed point. Assume now that (by some linearization) we can rewrite $f(x)$ in an equivalent form $g(x) - x$. Then finding a solution of $f(x) = 0$ amounts to finding a fixed point of g. Now, if the interval evaluation of g satisfies $g(\mathbf{x}) \subseteq \mathbf{x}$, by (1) we know that for all x in $\mathbf{x}$, $g(x) \in \mathbf{x}$, and Brouwer's theorem can be applied.

Parameterized Systems

As coefficients of equations often represent physical measurements, they are only known to lie within some intervals of confidence. So it is more significant to consider a parameterized system $f(p, x)$, where p denotes the set of parameters. The nice thing about interval theorems is that they can be extended to the parameterized case almost straightforwardly. This is as simple as plugging intervals in place of reals in the formulae. If $\mathbf{p}$ represents the domain of the parameters, then a "safe" box $\mathbf{x}$ ensures that

$$\big(\forall p \in \mathbf{p}\big)\big(\exists x \in \mathbf{x}\big) \mid f(p, x) = 0.$$

Some researchers focus today on situations where more freedom in the quantifiers is required. Given parameters p and q, one may rather look for a box $\mathbf{x}$ such that $\big(\forall x \in \mathbf{x}\big)\big(\forall p \in \mathbf{p}\big)\big(\exists q \in \mathbf{q}\big) \mid f(p, q, x) = 0$. Methods under development resort to a more complex algebraic structure called *generalized intervals*, where bounds are not constrained to be ordered (e.g., $[1, -1]$ is a valid interval).

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Active Contours for Segmentation: The Shape Gradient Approach

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Abstract. The variational approach to image or video segmentation consists in defining an energy depending on local or global image characteristics, the minimum of which being reached for objects of interest. This presentation focuses on energies written as a boundary or a domain integral. The shape gradient approach provides the derivative of the energy with respect to the domain. Some hypotheses are proposed to derive a practical expression. It allows to determine a contour velocity indicating a way to deform the domain in order to lower its energy. The shape gradient approach can be seen as a general framework for boundary-based and region-based segmentation.

1 Introduction

The variational approach to image or video segmentation consists in defining an energy depending on local or global image characteristics, the minimum of which being reached for objects of interest. This presentation focuses on energies written as an integral on a contour of a function independent of the contour or an integral on a domain of a function which can depend on this domain

$$E_{\text{boundary}}(\Omega) = \int_{\Gamma} \varphi(s) \; \mathrm{d}s \qquad E_{\text{region}}(\Omega) = \int_{\Omega} \phi(x, \Omega) \; \mathrm{d}x \qquad (1)$$

where Γ is the oriented boundary of Ω, φ is called object boundary descriptor, and ϕ is called object region descriptor.

2 Active Contours Based on Shape Gradient

Since there is no analytical expression of the optimal domain in general, an iterative minimization method should be considered. A gradient descent method, although not mandatory for boundary-based energies, allows to correctly deal with region-based energies. However, determining the derivative of the energy with respect to Ω by a calculus of variations can be complex. In the context of shape optimization, a general expression of this derivative, called shape derivative or shape gradient, was given

$$\begin{cases} \mathrm{d}E_{\text{boundary}}(\Omega, V) = \int_{\Gamma} \left(\frac{\partial \varphi(s)}{\partial N} - \varphi(s) \; \kappa(s) \right) \; V(s) \; \cdot N(s) \; \mathrm{d}s \\ \mathrm{d}E_{\text{region}}(\Omega, V) = \int_{\Omega} \mathrm{d}\phi(x, \Omega, V)\mathrm{d}x - \int_{\Gamma} \phi(s, \Omega) \; V(s) \; \cdot N(s) \; \mathrm{d}s \end{cases} \qquad (2)$$

where N is the inward unit normal of Γ, κ is the curvature of Γ, and V is the unknown, local velocity along Γ.

3   Toward an Expression Without Domain Integral

The expression of $\mathrm{d}E_{\mathrm{region}}$ is not readily usable because of the presence of the domain integral. In this presentation, two hypotheses are given to allow to derive a practical expression of the form

$$\mathrm{d}E_{\mathrm{region}}(\Omega, V) = -\int_{\Gamma} (\phi(s, \Omega) + \text{additional terms})\, V(s)\, \cdot N(s)\, \mathrm{d}s \quad . \qquad (3)$$

Then, the velocity V can easily be chosen such that the shape gradient is negative, leading to the following evolution equation

$$\frac{\partial \Gamma}{\partial \tau}(s, \tau) = V_{\tau}(s) = [\phi(s, \Omega_{\tau}) + \text{additional terms}]\, N_{\tau}(s) \quad , \qquad (4)$$

typical of active contours. This velocity is not the unique choice. Other choices, for a use of the shape gradient based on a different point of view, can be made and an example is given in the case of object tracking in a video sequence.

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Notes

Les séminaires doctorants

Les séminaires des doctorants STIC permettent aux futurs docteurs d'échanger leurs expériences dans leur travail de thèse, tant sur le plan scientifique que sur le plan professionnel et éducatif. Ces rencontres ont lieu mensuellement dans l'un des laboratoires STIC de Sophia Antipolis.

Un séminaire est l'occasion de quatre interventions, trois effectuées par des non permanents et une par un jeune permanent. Chaque intervention comporte un exposé technique d'une vingtaine de minutes et une période d'échanges et de retours d'expérience d'une dizaine de minutes.

Ces actes compilent les résumé en anglais des exposés techniques du séminaire doctorant du 11 avril 2006.

L'ADSTIC

L'ADSTIC est l'association des doctorants du campus sciences et techniques de l'information et de la communication de l'université de Nice Sophia Antipolis. Créée en 2004, l'ADSTIC est une association loi 1901.

Notre but essentiel est de faciliter les contacts entre les doctorants des différentes disciplines présentes sur le campus STIC, de les informer et de valoriser leur formation doctorale. L'ADSTIC se veut aussi un lien entre les doctorants passés, actuels et futurs...

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